Black Holes in Bulk Viscous Cosmology

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Abstract We investigate the effects of the accretion of phantom energy with non-zero bulk viscosity onto a Schwarzschild black hole and show that black holes accreting viscous phantom energy will lose mass rapidly compared to the non-viscous case. When matter is incorporated along with the phantom energy, the black holes meet with the same fate as bulk viscous forces dominate matter accretion. If the phantom energy has large bulk viscosity, then the mass of the black hole will reduce faster than in the small viscosity case.

Keywords Accretion · Black hole · Bulk viscosity · Phantom energy

1 Introduction

Observations of WMAP [1–3] and supernova of type Ia data [4] have revealed that our Universe is filled with an exotic dark energy apart from dark matter. The nature and composition of this energy is still an open problem but its dynamics is well understood i.e. it causes an approximately exponential expansion of the Universe (see [5–12] for recent reviews on dark energy). Astrophysical data suggest that about two thirds of the critical energy density is stored in the dark energy component. For the equation of state (EoS) parameter $\omega < -1$, the fluid is called phantom energy (PE). Observations show that ω is constrained in the range $-1.38 < \omega < -0.82$ [13], thus providing evidence of phantom energy in the Universe. The PE violates all the energy conditions in all forms (weak, null, strong or dominant). The phantom energy can cause some peculiar phenomena e.g. the existence of wormholes [14,

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15], infinite expansion of the Universe in a finite time causing a Big Rip (BR) and the destruction of all gravitationally bound structures including black holes [16–19]. In particular, black holes will continuously lose mass and disappear near the BR (see [20, 21] for the opposite viewpoint).

Dark energy with bulk viscosity has a peculiar property to cause accelerated expansion of phantom type in the late evolution of the universe [22–24]. It can also alleviate several cosmological puzzles like cosmic age problem [25], coincidence problem [26] and phantom crossing [27]. We will consider phantom energy as an imperfect fluid, implying that the PE could contain non-zero bulk and shear viscosities [28]. The bulk viscosities are negligible for non-relativistic and ultra-relativistic fluids but are important for the intermediate cases. In viscous cosmology, shear viscosities arise in relation to space anisotropy while the bulk viscosity accounts for the space isotropy [22–24, 29]. Generally, shear viscosities are ignored (as the CMB does not indicate significant anisotropies) and only bulk viscosities are taken into account for the fluids in the cosmological context. Moreover, bulk viscosity related to a grand unified theory phase transition may lead to an explanation of the accelerated cosmic expansion [30].

Babichev et al. [17] studied the effects of the accretion of phantom energy onto a Schwarzschild black hole taking PE to be a perfect fluid. As a first order approximation, the bulk viscosity can be ignored, but to get a better picture we need to incorporate it into the phantom fluid. We have adopted the procedure of [17, 31] for our calculations.

The plan of the paper is as follows: in the next section we review viscous cosmology; in section three we discuss the relativistic model of accretion onto a black hole; in the subsequent section we use results from viscous cosmology for the accretion model; next we give two examples to illustrate the accretion process with a constant and power law viscosity. In section six we study black hole evolution in the presence of matter and viscous phantom energy. Finally we conclude the paper with a brief discussion of our results.

2 Bulk-Viscous Cosmology

We assume the background spacetime to be homogeneous, isotropic and spatially flat ($k = 0$) and described by the Friedmann-Robertson-Walker (FRW) metric given by

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta \, d\varphi^2)], \tag{1}$$

where $a(t)$ is the scale factor. We also assume that the spacetime is filled with only one component fluid i.e. the viscous phantom energy of energy density ρ (however, in section six, we shall incorporate matter along with phantom energy). The Einstein field equations for the FRW-metric (in the units $c = 1 = 8\pi G$) are

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho, \tag{2}$$

and

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{1}{6}(\rho + 3p), \tag{3}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter, p is the effective pressure containing the isotropic pressure p_{pe} and the bulk viscous pressure p_{vis}, given by

$$p = p_{pe} + p_{vis}. \tag{4}$$

Here $\rho = \rho_{\rm pe} + \rho_{\rm vis}$ and $p_{\rm vis} = -\xi u^{\mu}_{;\mu}$, where u^{μ} is the velocity four vector and $\xi = \xi(\rho_{\rm vis}, t)$ is the bulk viscosity of the fluid [32]. Equation (4) shows that negative pressure due to viscosity contributes in the effective pressure which causes accelerated expansion. In the FRW model, the expression $u^{\mu}_{;\mu} = 3\dot{a}/a$ holds. Also, ξ is generally taken to be positive in order to avoid the violation of second law of thermodynamics [33].

The energy conservation equation is

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{5}$$

Assume that the viscous fluid equation of state (EoS) is

$$p = \omega\rho = (\gamma - 1)\rho. \tag{6}$$

Note that if $\gamma = 0$ (or $\omega = -1$), (6) represents the EoS for cosmological constant. Furthermore if $\gamma < 0$, it represents phantom energy. In general, for normal matter $1 \leq \gamma < 2$.

Using (2)–(6), we get the equation governing the evolution of $H(t)$ for a given ξ as

$$2\dot{H} + 3\gamma H^2 - 3\xi H = 0. \tag{7}$$

On integration, (7) gives

$$H(t) = \frac{\exp\{\frac{3}{2}\int \xi(t)dt\}}{C + \frac{3}{2}\gamma \exp\{\frac{3}{2}\int \xi(t)dt\}}, \tag{8}$$

where C is a constant of integration. Note that (8) can further be solved to get the evolution of $a(t)$ as

$$a(t) = D\left(C + \frac{3}{2}\gamma \int \exp\left\{\frac{3}{2}\int \xi(t)dt\right\}dt\right)^{\frac{2}{3\gamma}}, \tag{9}$$

where D is a constant of integration. Thus for a given value of ξ we can obtain expressions of $a(t)$, $\rho(t)$ and $p(t)$ from the system of (5)–(9).

3 Accretion onto Black Hole

In the background of FRW spacetime, we consider, as an approximation, a gravitationally isolated Schwarzschild black hole (BH) of mass M whose metric is specified by the line element:

$$ds^2 = -\left(1 - \frac{M}{4\pi r}\right)dt^2 + \left(1 - \frac{M}{4\pi r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{10}$$

The background spacetime is assumed to contain one test fluid, namely the phantom energy with non-vanishing bulk viscous stress $p_{\rm vis}$. The fluid is assumed to fall onto the BH horizon in the radial direction only which is in conformity with the spherical symmetry of the BH. Thus, the velocity four vector of the phantom fluid is $u^{\mu} = (u^t(r), u^r(r), 0, 0)$ which satisfies the normalization condition $u^{\mu}u_{\mu} = -1$. This phantom fluid is specified by the stress energy tensor for a viscous fluid [28, 33]:

$$T^{\mu\nu} = (\rho + p)u^{\mu}u^{\nu} + pg^{\mu\nu}. \tag{11}$$

Using the energy momentum conservation for $T^{\mu\nu}$, we get

$$ur^2 M^{-2}(\rho + p)\sqrt{1 - \frac{M}{4\pi r} + u^2} = C_1, \tag{12}$$

where $u^r = u = dr/ds$ is the radial component of the velocity four vector and C_1 is a constant of integration. The second constant of motion is obtained by contracting the velocity four vector of the phantom fluid with the stress energy tensor $u_\mu T^{\mu\nu}_{;\nu} = 0$, which gives

$$ur^2 M^{-2} \exp\left[\int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho' + p(\rho')}\right] = -A, \tag{13}$$

where A is a constant of integration. Also ρ_h and ρ_∞ are the energy densities of the phantom fluid at the horizon of the BH, and at infinity respectively. From (12) and (13) we have

$$(\rho + p)\sqrt{1 - \frac{M}{4\pi r} + u^2} \exp\left[-\int_{\rho_\infty}^{\rho_h} \frac{d\rho'}{\rho' + p(\rho')}\right] = C_2, \tag{14}$$

with $C_2 = -C_1/A = \rho_\infty + p(\rho_\infty)$. In order to calculate the rate of change of mass of black hole $\dot{M}$, we integrate the flux of the bulk viscous phantom fluid over the entire BH horizon to get

$$\dot{M} = \oint T^r_t dS. \tag{15}$$

Here T^r_t determines the energy momentum flux in the radial direction only and $dS = \sqrt{-g} d\theta d\varphi$ is the infinitesimal surface element of the BH horizon. Using (12)–(15), we get

$$\frac{dM}{dt} = \frac{AM^2}{16\pi}(\rho + p), \tag{16}$$

which clearly demonstrates the vanishing mass of the black hole if $\rho + p < 0$. Integration of (16) leads to

$$M = M_0\left(1 - \frac{t}{\tau}\right)^{-1}, \tag{17}$$

where M_0 is the initial mass of the black hole and modified characteristic accretion time scale $\tau^{-1} = [\frac{AM_0}{16\pi}\{(\rho_{pe} + p_{pe}) - \frac{3\xi}{t}\ln(\frac{a}{a_0})\}]$, a_0 being the value of the scale factor at time t_0. Note that during the integration of (16), we assumed ρ_{pe} and p_{pe} to be constants. In the coming subsections, we shall take these as time dependent entities.

4 Accretion of Viscous Phantom Fluid

We now study the BH mass evolution in two special cases: (a) constant viscosity; and (b) power law viscosity.

4.1 Constant Bulk Viscosity

For constant viscosity $\xi = \xi_o$, the evolution of $a(t)$ is determined by using (9). It gives

$$a(t) = a_0\left[1 + \frac{\gamma H_o B(t)}{\xi_o}\right]^{\frac{2}{3\gamma}}, \tag{18}$$

where

$$B(t) \equiv \exp\left(\frac{3t\xi_o}{2}\right) - 1. \tag{19}$$

Using (5), (6) and (8) the density evolution is given by

$$\rho(t) = \frac{\rho_o \exp(3\xi_o t)}{[1 + \frac{\gamma H_o B(t)}{\xi_o}]^2}. \tag{20}$$

Here $\rho_o = 3H_o^2$. Further, for $\gamma < 0$ the BR singularity occurs in a finite time at

$$\tau = \frac{2}{3\xi_o} \ln\left(1 - \frac{\xi_o}{H_o \gamma}\right). \tag{21}$$

Finally, the BH mass evolution is determined by solving (16) and (20) to get

$$M = M_0 \left[1 - \frac{AM_0}{8\pi\gamma}\left(\frac{\xi_o}{\Delta} - 1\right)(\xi_o - \gamma H_o)\right]^{-1}, \tag{22}$$

where

$$\Delta \equiv \xi_o + (-1 + e^{\frac{3t\xi_o}{2}})\gamma H_o. \tag{23}$$

This mass is displayed for different values of viscosity at different times in Table 1.

It is apparent from Table 1 that for a fixed viscosity, the mass ratio decreases with time implying that mass of black hole is decreasing for an initial mass. Similarly, at any given time, the mass ratio also decreases with the increase in viscosity. Thus the greater the value of viscosity parameter, the greater would be its effects on the BH mass.

4.2 Power Law Viscosity

If the viscosity has power law dependence upon density i.e. $\xi = \alpha \rho_{\text{vis}}^s$, where α and s are constant parameters, it has been shown [35, 36] that it yields cosmologies with a BR if $\sqrt{3\alpha} > \gamma$ and $s = 1/2$. Thus we take $\xi = \alpha\rho^{\frac{1}{2}}$ as a special case. Then the scale factor evolves as

$$a(t) = a_0 \left(1 - \frac{t}{\tau}\right)^{\frac{2}{3(\gamma - \sqrt{3\alpha})}}. \tag{24}$$

Table 1 The mass ratio M/M_0 of black hole for different choices of constant viscosity ξ_o. The initial mass is, throughout, taken to be $50M_\odot$ or 10^{32} kg

$t \downarrow \xi \rightarrow$	$\xi_1 = 10^{-17}$	$\xi_2 = 10^{-18}$	$\xi_3 = 10^{-19}$	$\xi_4 = 10^{-20}$
$t_1 = 10^7$	3.43427×10^{-4}	2.44662×10^{-3}	6.31285×10^{-3}	7.49184×10^{-3}
$t_2 = 10^{10}$	3.43544×10^{-7}	2.45261×10^{-6}	6.35248×10^{-6}	7.55357×10^{-6}
$t_3 = 10^{13}$	3.43516×10^{-10}	2.45258×10^{-9}	6.35247×10^{-9}	7.55358×10^{-9}
$t_4 = 10^{17}$	1.23994×10^{-14}	2.10182×10^{-13}	5.86096×10^{-13}	7.01997×10^{-13}

Table 2 The mass ratio M/M_0 of black hole for different choices of equation of state. The initial mass is $50M_\odot$ or 10^{32} kg

$t \downarrow \gamma \rightarrow$	$\gamma_1 = -1 \times 10^{-1}$	$\gamma_2 = -2 \times 10^{-1}$	$\gamma_3 = -3 \times 10^{-1}$	$\gamma_4 = -4 \times 10^{-1}$
$t_1 = 10^{10}$	4.71915×10^{-5}	2.35963×10^{-5}	1.5731×10^{-5}	1.17983×10^{-5}
$t_2 = 10^{13}$	4.79136×10^{-8}	2.35968×10^{-8}	1.57312×10^{-8}	1.17984×10^{-8}
$t_3 = 10^{17}$	4.66492×10^{-12}	2.30523×10^{-12}	1.51867×10^{-12}	1.12539×10^{-12}
$t_4 = 10^{20}$	4.97349×10^{-14}	5.20946×10^{-14}	5.28811×10^{-14}	5.32744×10^{-14}

The density of phantom fluid evolves as

$$\rho(t) = \frac{4}{3\tau^2(\gamma - \sqrt{3}\alpha)^2}\left(1 - \frac{t}{\tau}\right)^{-2}, \tag{25}$$

or in terms of critical density ρ_{cr} as

$$\rho(t) = \rho_{cr}\left(1 - \frac{t}{\tau}\right)^{-2}. \tag{26}$$

The corresponding BR time τ is given by

$$\tau = \frac{2}{3(\sqrt{3}\alpha - \gamma)}H_o^{-1}. \tag{27}$$

Finally, the mass evolution of BH is determined by using (16) and (25) is

$$M = M_0\left[1 + \frac{AM_0}{4\pi(\sqrt{3}\alpha - \gamma)}\frac{t}{\tau(\tau - t)}\right]^{-1}. \tag{28}$$

Note that when $\alpha = 0$, this case reduces to that of Babichev et al. [17]. The mass in (28) in displayed for different values of EoS parameter γ at different times in Table 2 and displayed graphically in Fig. 1. As shown, the mass decreases gradually with the decrease in the EoS parameter γ. Note that we have not graphically displayed the mass for different viscosities given in Table 1 because the variation is not significantly different for most time scales.
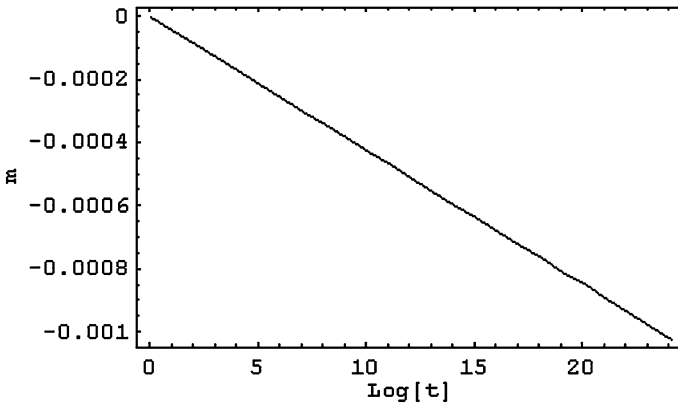
5 Examples

We now solve examples to demonstrate the accretion of viscous phantom energy onto a BH. The formalism is adapted from [17].

5.1 Viscous Linear EoS

We choose the viscous linear EoS, $p = \omega\rho_{pe} - 3H\xi_o$ with $\omega < -1$. The ratio of the number densities of phantom fluid particles at the horizon and at infinity is given by

$$\frac{n(\rho_h^{pe})}{n(\rho_\infty^{pe})} = \left[\frac{\rho_h^{pe}(1 + \omega) - 3\xi_o H}{\rho_\infty^{pe}(1 + \omega) - 3\xi_o H}\right]^{\frac{1}{(1+\omega)}}. \tag{29}$$

Fig. 1 For an initial mass of black hole $M_0 = 10^{32}$ kg, the evolution of the mass parameter $m = (M/M_0) - 1$ is plotted against the logarithmic time with $\alpha = 10^{-5}$ and $t_H = 10^{17}$ s

The critical points of accretion (the point where the speed of fluid flow becomes equal to the speed of sound i.e. $u_*^2 = c_s^2$) are given by

$$u_*^2 = \frac{\omega}{1 + 3\omega}; \qquad x_* = \frac{1 + 3\omega}{2\omega}. \tag{30}$$

The constant A appearing in (16) is determined to be

$$A = \frac{|1 + 3\omega|^{\frac{1+\omega}{2\omega}}}{4|\omega|^{3/2}}. \tag{31}$$

Notice that the constant A is the same as for the non-viscous case [17]. Also, the density of phantom energy at the horizon is given by

$$\rho_h^{pe} = \frac{3\xi_o H}{1 + \omega} + \left(\frac{4}{A}\right)^{\frac{\omega-1}{\omega+1}} \left(\rho_\infty - \frac{3\xi_o H}{1 + \omega}\right). \tag{32}$$

Moreover, the speed of flow at the horizon is

$$u_h = -\left(\frac{A}{4}\right)^{\frac{\omega}{(\omega+1)}}. \tag{33}$$

The speed is negative as it is directed towards the BH. Also, the characteristic evolution time scale of the BH is given by

$$\tau^{-1} = 4\pi M_0 \frac{(1 + 3\omega)^{\frac{1+\omega}{2\omega}}}{4\omega^{3/2}} \left\{\rho_\infty^{pe}(1 + \omega) - \frac{3\xi_o}{t} \ln\left(\frac{a}{a_0}\right)\right\}. \tag{34}$$

Finally, substituting (34) in (17) we get the mass evolution of a BH in bulk viscous cosmology

$$M = M_0 \left[1 - 4\pi M_0 t \frac{(1 + 3\omega)^{\frac{1+\omega}{2\omega}}}{4\omega^{3/2}} \left\{\rho_\infty^{pe}(1 + \omega) - \frac{3\xi_o}{t} \ln\left(\frac{a}{a_0}\right)\right\}\right]^{-1}. \tag{35}$$

Since $\rho_\infty^{\mathrm{pe}}$ is unknown for our purpose, we have not evaluated M for different times numerically for tabular and graphical presentation.

5.2 Viscous Non-linear EoS

We here choose the EoS, $p = \omega \rho_{\mathrm{pe}} - 3H\xi(\rho_{\mathrm{vis}})$ with $\omega < -1$, where $\xi(\rho_{\mathrm{pe}}) = \alpha \rho_{\mathrm{pe}}^s$ with α and s are constants. The ratio of number densities is given by

$$\frac{n(\rho_h^{\mathrm{pe}})}{n(\rho_\infty^{\mathrm{pe}})} = \left(\frac{\rho_h}{\rho_\infty}\right)^{\frac{s}{(s-1)(1+\omega)}} \left(\frac{\rho_\infty(1+\omega) - 3H\alpha\rho_\infty^s}{\rho_h(1+\omega) - 3H\alpha\rho_h^s}\right) \tag{36}$$

The constant A appearing in (16) is determined to be

$$A = \left|\left(\frac{\rho_h}{\rho_\infty}\right)^{\frac{2s}{(s-1)(1+\omega)}} \left(\frac{\rho_\infty(1+\omega) - 3H\alpha\rho_\infty^s}{\rho_h(1+\omega) - 3H\alpha\rho_h^s}\right)^3\right|. \tag{37}$$

The speed of flow at the horizon becomes

$$u_h = -\left(\frac{\rho_h}{\rho_\infty}\right)^{\frac{2s}{(s-1)(1+\omega)}} \left(\frac{\rho_\infty(1+\omega) - 3H\alpha\rho_\infty^s}{\rho_h(1+\omega) - 3H\alpha\rho_h^s}\right)^2. \tag{38}$$

The critical points of accretion are given by

$$u_*^2 = \frac{\omega - 3s\alpha\rho_h^{s-1}}{1 + 3(\omega - 3s\alpha\rho_h^{s-1})}; \qquad x_* = \frac{1 + 3(\omega - 3s\alpha\rho_h^{s-1})}{2(\omega - 3s\alpha\rho_h^{s-1})}. \tag{39}$$

The characteristic evolution time scale τ is given by

$$\tau = \left[4\pi M_0 \left(\frac{\rho_h}{\rho_\infty}\right)^{\frac{2s}{(s-1)(1+\omega)}} \left(\frac{\rho_\infty(1+\omega) - 3H\alpha\rho_\infty^s}{\rho_h(1+\omega) - 3H\alpha\rho_h^s}\right)^3 \left\{\rho_\infty(1+\omega) - 3\frac{\alpha\rho_\infty^s}{t}\ln\left(\frac{a}{a_0}\right)\right\}\right]^{-1}. \tag{40}$$

Finally, using (40) in (17), the BH mass evolution is given by

$$M = M_0\left[1 - 4\pi M_0 t\left(\frac{\rho_h}{\rho_\infty}\right)^{\frac{2s}{(s-1)(1+\omega)}} \left(\frac{\rho_\infty(1+\omega) - 3H\alpha\rho_\infty^s}{\rho_h(1+\omega) - 3H\alpha\rho_h^s}\right)^3\right.$$

$$\left. \times \left\{\rho_\infty(1+\omega) - 3\frac{\alpha\rho_\infty^s}{t}\ln\left(\frac{a}{a_0}\right)\right\}\right]^{-1}. \tag{41}$$

As before, ρ_∞ is unknown, but further ρ_h is also unknown. As such, we again do not provide a tabular or graphical presentation.

6 Black Holes Accreting Both Matter and Viscous Phantom Fluid

We now consider a two component fluid, the viscous dark energy and matter. The matter part may be composed of both baryonic and non-baryonic matter. It is taken to be a perfect fluid while the PE is taken as a bulk viscous fluid. The effective pressure is represented by (4). The corresponding Einstein field equations (EFE) for the two component fluid become:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = T_{\mu\nu} + T_{\mu\nu}^{\mathrm{m}}. \tag{42}$$

The stress-energy tensor representing the two component fluid is given by

$$T^{\mu\nu} = (\rho + p + \rho_{\mathrm{m}})u^{\mu}u^{\nu} + pg^{\mu\nu}. \tag{43}$$

Here ρ_{m} is the energy density of the pressureless matter. Energy conservation holds independently for both fluids:

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{44}$$

$$\dot{\rho}_{\mathrm{m}} + 3H\rho_{\mathrm{m}} = 0. \tag{45}$$

Integrating (45), we have

$$\rho_{\mathrm{m}} = \rho_{\mathrm{m}_0}a^{-3}, \tag{46}$$

where $\rho_{\mathrm{m}_0} = \rho_{\mathrm{m}}(t_0)$. Similarly, integrating (44) leads to

$$\rho = \rho_{\mathrm{m}}\left[\left(\Xi + \frac{K}{3}a^{3/2}\right)^2 - 1\right], \tag{47}$$

where Ξ is a constant and K is given by

$$K = \frac{3\sqrt{3}\xi_o}{\sqrt{\rho_{\mathrm{m}_0}}}, \tag{48}$$

Thus the total energy density of the two component fluid is given by [34]

$$\rho \equiv \rho + \rho_{\mathrm{m}} = \rho_{\mathrm{m}_0}a^{-3}\left(\Xi + \frac{K}{3}a^{3/2}\right)^2. \tag{49}$$

Using (45) in (16) the evolution of black hole mass is given by

$$M = M_0\left[1 - 4\pi AM_0\left[\frac{\gamma\rho_{\mathrm{m}_0}}{H(t)}\left\{\frac{K^2}{9}\ln\left(\frac{a}{a_0}\right) - \frac{\Xi}{9a^3}(3\Xi + 4a^{3/2}K)\right.\right.\right.$$
$$\left.\left.\left. + \frac{\Xi}{9a_0^3}(3\Xi + 4a_0^{3/2}K)\right\}\right]\right]^{-1}, \tag{50}$$

where the scale factor $a(t)$ evolves as

$$a(t) = \left[\frac{3}{K}(e^{\frac{K}{2}\sqrt{\rho_{\mathrm{m}_0}/3}t + D_1} - \Xi)\right]^{2/3}, \tag{51}$$

and D_1 is the constant of integration determined by choosing $t = 0$ to get

$$D_1 = \frac{2}{K}\ln\left(\frac{K}{3}a_0^{3/2} + \Xi\right). \tag{52}$$

As pointed out in the next section, we cannot correctly discuss a BR scenario. However we can take a spacetime approximating it sufficiently earlier than the BR. We can than see its asymptotic behavior. when the scale factor shoots to infinity, the three terms in (50) will contribute significantly in the BH mass evolution. The mass will decrease by the accretion of PE ($\gamma < 0$) due to its strong negative pressure and is manifested in (50). Notice that the
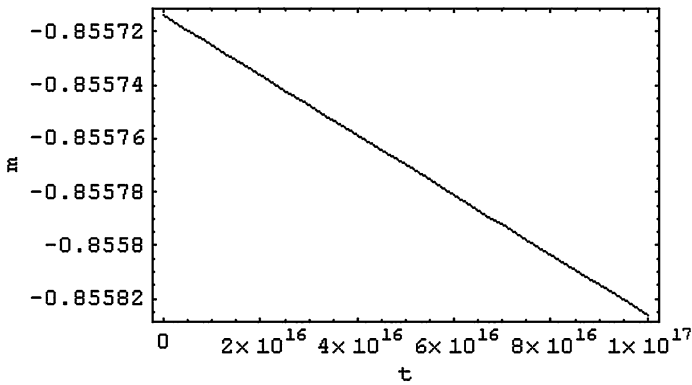
Fig. 2 For an initial mass of black hole $M_0 = 10^{32}$ kg, the evolution of m is plotted against the time parameter t with $A = 1/3$, $\Xi = 3$, $\xi_o = 10^{-16}$ kg m^{-1} s^{-1} and $\gamma = -10^{-1}$ while $H \approx 2.33 \times 10^{-18}$ m
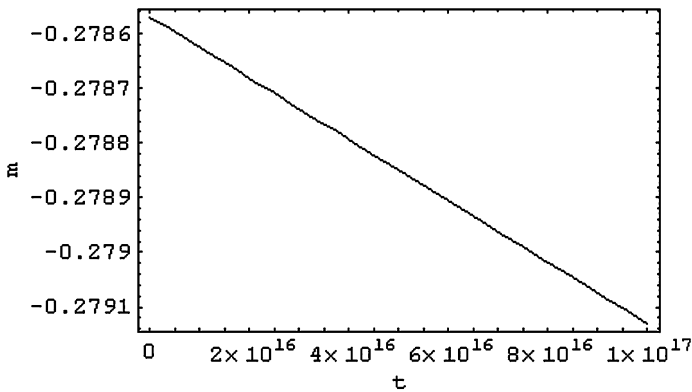
Fig. 3 For an initial mass of black hole $M_0 = 10^{32}$ kg, the evolution of m is plotted against the time parameter t with $A = 1/3$, $\Xi = 3$, $\xi_o = 10^{-16}$ kg m^{-1} s^{-1} and $\gamma = -2 \times 10^{-1}$ while $H \approx 2.33 \times 10^{-18}$ m

final expression for BH mass depends only on the initial matter density ρ_{m_0} in addition to constant bulk viscosity ξ_o. The corresponding behavior of BH mass evolution is shown in Figs. 2 and 3 for different values of model parameters. Thus for a shift of parameter γ by 2, yields in the decrease of mass ratio by a factor of 2. The decrease in the mass of the BH is observed with time showing that phantom energy accretion will be dominant over matter accretion.

7 Conclusion and Discussion

We have analyzed the accretion of bulk viscous phantom energy onto a BH. The modeling is based on the relativistic model of accretion for compact objects. The viscosity effects in cosmology are used to give an alternative to cosmic accelerated expansion other then dark energy and quintessence. The evolution of BHs in such a Universe accreting viscous phantom energy would result in a gradual decrease in mass. This gradual decrease would be faster than the non-viscous case [17] due to additional terms containing viscosities coupled

with mass. Lastly, it is shown that BHs accreting both matter and viscous PE will also meet with the same fate as the viscous forces dominate over the matter component for sufficiently large scale factor $a(t)$.

From this analysis, we can draw the conclusion that PE containing viscous stresses can play a significant role in the BH mass evolution if the viscosity is sufficiently high for an appropriate EoS. Though the viscous stresses are negligibly small $O(10^{-8}\ \text{N s m}^{-2})$ at the local scale of space and time they can play a significant role in time scales of $\sim$ Gyrs. The higher the viscosity of the phantom fluid, the sharper the decrease in the BH mass. BHs of all masses, ranging from the solar mass to the intermediate mass to the supermassive, will all meet the same fate.

As an extension to this problem, it is interesting to study the accretion of the phantom fluid onto primordial BHs that had formed due to initial density fluctuations in the primordial plasma. The mini-primordial BHs evaporating now via Hawking radiation would have a different initial mass and hence abundance than the standard scenario expects. This work is reported in a separate paper [37].

Notice that we have used the Friedmann model which is represented by an asymptotically curved spacetime and at the same time the Schwarzschild black hole, which is asymptotically flat. This may seem contradictory. Schwarzschild black hole has been dealt with in the context of closed Friedmann cosmology [38–40]. Any global problem in approximating the full situation by a Schwarzschild black hole inserted into Friedmann model arise near the big bang or the big crunch, defined in terms of the York time [41] as shown elsewhere [42], the effect will be at extremely late times in terms of the usual time parameter. More complete analysis of the asymptotic behavior near a singularity is also available [43], as such if we stay near to a singularity in spacetime, the approximation will be extremely good. Consequently our analysis will be satisfactory for black holes formed well after the big bang greater then 10^{-40} s and of the Big Rip (presumably much more before 10^{-40} s the rip). It is clear that we are unable to say whether there would/would not be a Big Rip as our analysis excludes it.

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